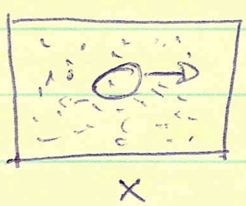
d.) Fluctuations in Plasma and the Test Particle Model

Contents

(a) Basic Ideas - Equilibrium Fluctuations

→ plasma:
- $1/n\lambda_D^3 \ll 1$ ⇒ many particles in Debye sphere
- $k_B T \gg e^2/r$ ⇒ thermal energy dominates electrostatic energy

→ Equilibrium : Balances
→ absorption vs. emission
(equivalent) → fluctuation vs. dissipation

i.e. v  ① emission: discrete particle in Vlasov fluid emits waves

$$\underline{\nabla} \cdot \underline{D} = \underline{\nabla} \cdot \underline{\underline{\epsilon}}\, \underline{E} = 4\pi n_0 q\, \delta(\underline{x} - x(t))$$

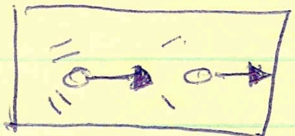
i.e. → a la' boat wake on water
→ Cerenkov emission

∴ discrete emission ⇔ fluctuation

i.e. - particle kinetic energy coupled to wave energy
- Cerenkov emission ⇔ particle slowing down

② Cerenkov emitted waves damp via → Landau damping
→ in/on Vlasov fluid (i.e. Landau damping for $\nu \to 0$)

i.e.



emitted waves damp → $\left\{ \begin{array}{l} \text{absorption} \\ \text{dissipation} \end{array} \right\}$ heats particles

So, conceptual picture of thermal equilibrium fluctuations is detailed balance of:

→ Cerenkov emission of waves from individual discrete particles

→ absorption of waves via Landau damping on Vlasov fluid

NB: ① Here, assume periodic B.C.'s → no radiative damping, outgoing waves, etc.

Note in this picture, each particle plays a dual role (i.e. "double agent"):

② In general, take damping length finite

i.e. $l_{damp} \sim \frac{\omega}{k} / |\gamma_k| < L_{system}$

As:

- "emitter": a <u>discrete</u> particle moving along some specified (unperturbed) orbit

i.e.  an identifiable 'pea' in a 'pea soup' composed of other peas.

- "absorber": an element of the Vlasov <u>fluid</u> responding to, and Landau damping, emission from (other) discrete particles

i.e.  a "crushed pea" element of the "pea soup" of the Vlasov fluid.

<u>so</u>

→ equilibrium plasma ≡ soup/gas of 'dressed' test particles

↓

<u>Vlasov fluid</u> screening

⇒ "test particle model"

→ every pea in the soup acts like soup for all the other peas ....

Note: Useful Analogy

	Brownian Motion	Equilibrium Plasma
Fluctuation	$\langle \tilde{v}^2 \rangle_\omega$	$\langle \tilde{E}^2 \rangle_{k,\omega}$
Exciton	ω-mode	$k, \omega(k) \rightarrow$ plasma waves
Emission/Source	$\langle \tilde{f}^2 \rangle$ – fluid thermal force	particle discreteness
Absorption/Damping	$\beta \leftrightarrow$ Stokes Drag on Particle	$\epsilon_{IM} \leftrightarrow$ Landau Damping of Collective Modes

n.b. : "Brownian Motion"

$$\frac{d\tilde{v}}{dt} + \beta \tilde{\underline{v}} = \frac{\tilde{f}}{m}$$

ii) Test Particle Model – Fluctuation Spectrum

→ As noted before, basic idea is that:

- each particle both a 'discrete emitter' and participant in Vlasov fluid screening cloud

- fluctuations weak → unperturbed orbits valid.

if consider stationary cons:

$$\delta f = f^c + \tilde{f}$$

f^c ↓ coherent Vlasov response (screening)

$\tilde{f}$ ↓ discrete particle source

(cf. Debye calculation)

$$\delta f = \frac{\frac{|e|}{m} \tilde{E}_{k,\omega}\, \partial\langle f\rangle/\partial v}{-i(\omega - kv)} + |e|\, \delta(x - x(t))\, \delta(v - v(t))$$

↓ coherent response

↓ discreteness source.

$$\nabla^2\phi = 4\pi n_0 |e| \int dv\, \delta f = 4\pi n_0 |e| \int dv\, f^c + 4\pi n_0 |e| \int dv\, \tilde{f}$$

$$\Rightarrow \quad \vec{\phi}_{k,\omega} = \frac{4\pi n_0 |e|}{k^2} \int dv\, \tilde{f} \Big/ \epsilon(k,\omega)$$

$$\epsilon(k,\omega) = 1 + \frac{\omega_p^2}{k} \int dv\, \frac{\partial \langle f \rangle / \partial v}{\omega - kv}$$

using u.p.o. :

$$\int dv\, \tilde{f}_k = \int dx\, e^{-ikx} |e|\, \delta(x - x(t))$$
$$= |e|\, e^{-ikvt}$$

<u>so</u>

$$\vec{\phi}_k(t) = \underline{\epsilon}^{-1}(k,t)\, \frac{4\pi n_0 |e|}{k^2}\, e^{-ikvt}$$

Note: strictly speaking, have :

$$\underline{\epsilon}(k,t)\, \vec{\phi}_k(t) = \frac{4\pi n_0 |e|}{k^2}\, e^{-ikvt}$$

$$\underline{\text{so}} \quad \phi_k(t) = \underline{\epsilon}^{-1}(k,t)\, \frac{4\pi n_0 |e|}{k^2}\, e^{-ikvt} + \phi_k^{\text{homog.}}\, e^{-i\omega_k t}$$

driven solution (discreteness)

homogeneous solution

$\omega_k = \omega_r(k) + i\omega_i(k)$ $\Rightarrow$ eigenmode freq.

Now, – time asymptotically ....

– for $\omega_i(k) < 0$ $\Rightarrow$ collective modes damped ...

$\Rightarrow$ only discreteness driven solutions persist ...

Catch: $\rightarrow$ For $\omega_i \lesssim 0$ $\Rightarrow$ i) need wait quite a long time..

ii) for sufficient source strength, amplification to nonlinearity occurs ....

n.b. moving toward, but not to, marginal stability $\Rightarrow$ $\tau_{relax} \rightarrow \infty$

$\rightarrow$ if unstable modes, require ultimate nonlinear damping to balance noise

i.e. $\epsilon_{IM} = \epsilon_{IM}(k, \omega, \langle \tilde{\phi}^2 \rangle)$ ...

"noise" = thermal + nonlinear, in that case

...

Proceeding, then test particle model $\Rightarrow$

$$\boxed{\langle\hat{\phi}^2\rangle_{k,\omega} = \left(\frac{4\pi n_0 |e|}{k^2}\right)^2 \int dv_1 \int dv_2 \, \frac{\langle \tilde{f}(1)\tilde{f}(2)\rangle_{k,\omega}}{|\epsilon(k,\omega)|^2}}$$

$\therefore$ all content $\rightarrow \langle \tilde{f}^2\rangle_{k,\omega}$ (abbreviation)

$\rightarrow \epsilon(k,\omega)$

Now, for discreteness noise:

$$\tilde{f} = \frac{1}{n}\sum_{i=1}^{N} \delta(x - x_i(t))\,\delta(v - v_i(t))$$

$\rightarrow$

$$\left.\begin{aligned} x_i(t) &= x_{i0} + v_i t \\ v_i(t) &= \text{const} \end{aligned}\right\} \text{u.p.o.}$$

$\rightarrow$ assume (discrete) uncorrelated test particles, so:

so $\langle\ \rangle = n \int d\underline{x_0} \int d\underline{v_i} \, \langle f(v_i)\rangle$

$\sim$ Maxwellian

i.e. simple avg. over equilibrium distribution

$(k_B T \gg e^2/\bar{r})$

so

$$\langle \tilde{f}(1)\tilde{f}(2)\rangle = n\int d\underline{x}_i \int d\underline{v}_i \left[\left(\frac{1}{n}\sum_{i=1}^{N} \delta(\underline{x}_1 - \underline{x}_i(t))\, \delta(\underline{v}_1 - \underline{v}_i(t)) \right) * \frac{1}{n}\sum_{j=1}^{N} \delta(\underline{x}_2 - \underline{x}_j(t))\, \delta(\underline{v}_2 - \underline{v}_j(t)) \right] \langle f \rangle$$

$$= \frac{1}{n}\, \delta(\underline{x}_1 - \underline{x}_2)\, \delta(\underline{v}_1 - \underline{v}_2) \langle f \rangle$$

as avg. vanishes unless $\begin{pmatrix} \underline{x}_i \\ \underline{v}_i \end{pmatrix} = \begin{pmatrix} \underline{x}_j \\ \underline{v}_j \end{pmatrix}$

N.B. : Uncorrelated test particles can only correlate with themselves...

so $$\boxed{\langle \tilde{f}(1)\tilde{f}(2)\rangle = \frac{\langle f \rangle}{n}\, \delta(\underline{x}_1 - \underline{x}_2)\, \delta(\underline{v}_1 - \underline{v}_2)}$$ Discreteness Correlation

See Pg. 11 for furthur details...